# Geometric Analysis of Forumad Mosques' Ornaments

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## Abstract

This article analyzes the architectural designs and ornaments of a 12th century structure in Iran. Friday Mosque of Forumad, *Masjid-i Jami' Forumad* is a prototype mosque in Persian architecture using an application of ornaments. Architectural ornaments are important examples of practical geometry. The brilliant stucco ornaments, elaborate strapwork patterns with turquoise and cobalt blue, and floral motifs with underlying geometric patterns, made this mosque a worthwhile building in early Iranian-Islamic period. By introducing the mosque's features, the ornaments that adorn some of the walls will be analyzed and geometric methods for constructing their underlying designs will be presented.

## 1 Introduction

Early mosques were simple buildings with no decoration. After some decades and under the influence of advanced civilizations of conquered territories, architectural ornaments entered into their existence. Since Islamic thought prohibited artists and craftsmen of creating human figures, they turned their attention to abstract forms. They also benefited from mathematics, which flourished in early Islamic period in Baghdad. This resulted to a widespread movement in Islamic geometric art. Even though the designers of those structures had remarkable knowledge of applied geometry, there exist few written records to explain various methods that they employed for constructions of their designs [7]. Nevertheless, based on the detailed and sophisticated geometric designs in these documents we may suggest that some degree of mathematical literacy may have existed among the master builders, architects and master engineers [1]. For creating ornamental designs, the traditional tools of medieval times, a compass, straightedge, and set square, may well have been used by those master builders versed in construction techniques [5]. The architectural elements for decoration created by the artists and artisans of the Seljuq period of 11th and 12th centuries gradually flourished in Iran, central Asia, and North Africa. At first, brickwork was a common approach for creating abstract motifs. Glazed brick gradually entered into architectural ornaments. Forumad was a pioneer in applying glazed brick in Iranian mosques. This structure is a small mosque with its ornaments decorated inside as well as outside walls. Following geometric substructures of ornamental designs in the Forumad mosque, we endeavor to illustrate practical geometry and try to detect design methods, which can be created by common instruments used by the designers and architects at that time.

## 2 Forumad Mosque

Friday Mosque of Forumad, *Masjid-i Jami' Forumad,* is a two-iwan mosque, which is located in Forumad village of Semnan Province in the Northeast of Iran. Iwan is a vaulted hall, walled on three sides, with one end entirely open. The four-iwan mosques that became one of the classical features of the religious architecture of Iran are in fact a continuation of two-iwan mosque style. This mosque is made up of two iwans facing each other on either side of a courtyard lined from west to east. Although this mosque has no inscription, it is believed by most archeologists that the architectural style of it is in the style of either 12$^{th}$ or early 13$^{th}$ century mosques comparing with other two-iwan mosques [4]. Nevertheless, the advanced stuccoworks for designs suggest that some improvements have occurred later perhaps during the Ilkhanid period of the 14$^{th}$ century. This building is covered by stuccowork on the inside and by brickwork on the outside. It has two iwans, each about 6.6 meters width. The northern iwan is 12 meters in height and the height of the southern one is 14 meters. There are two vaults on both sides of each iwan, which are connected to the naves. There are also two naves in the east and the west sides of the mosque. Their roofs and walls were destroyed and just three arches, 24 meters in width, have survived. There are exquisite patterns made on these arches and elegant stuccoworks are above them under each vault. Each pedestal has an inscription in stucco by calligraphy, which is different from one another [6].There are also two walls with arch-façade in brickworks with geometric patterns and glazed bricks in turquoise and cobalt blue on the northern iwan. Variety, elegance and complexity of the designs in the Forumad mosque were noticed by French archeologist and art historian, and late director of the Archeological Services of Iran, Andre Godard, who excavated this mosque in the nineteen twenties [4]. Its brilliant stucco, glazed brick with various geometric and floral ornaments, made this building an outstanding mosque in the early Islamic period of Iran, especially from ornament perspective.

**Figure 1:** *Forumad mosque(see also more images at the end of the article).*

## 3 Analyze of the Architectural Ornaments

By depicting underlying designs of motifs made on the walls of the Forumad mosque in this section, we will analyse repetitive units, axes of symmetry, vectors, which transfer motifs, and design methods used in architectural ornaments of this building. All geometric patterns and designs have been created by the authors.

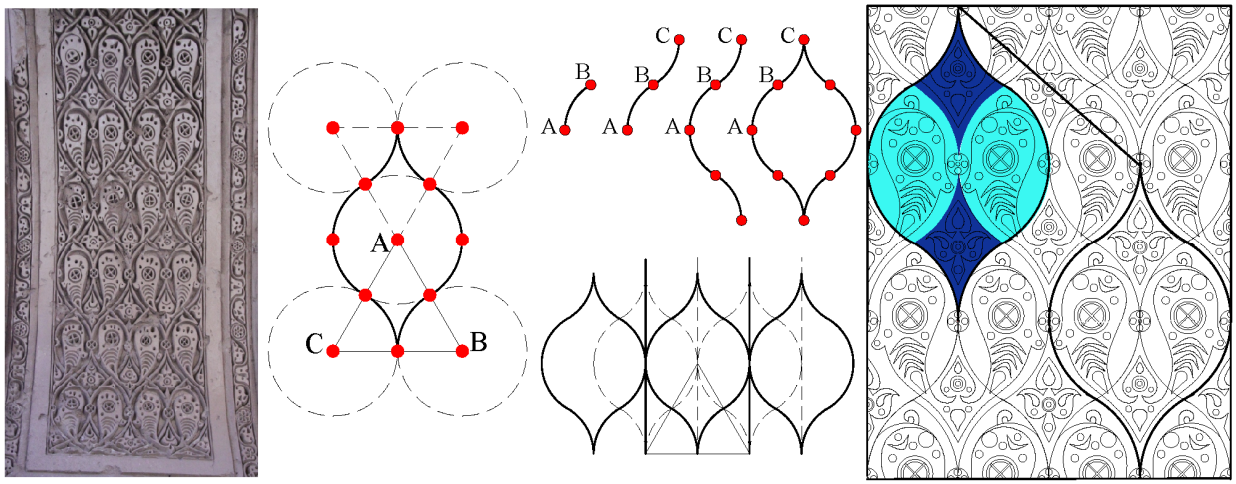


**Figure 2:** *Left: stucco in the eastern Iwan. Right: analysis of substructure.*

The very left image in Figure 2 is a photograph from a wall of the Forumad mosque. The floral motif of this pattern is created based on the properties of circles. Draw the equilateral triangle *ABC*. Then make three circles with centers at the of triangles' vertices and by a radius congruent to one half of a side of this triangle. So, the three circles meet each other at the midpoints of the triangles' sides. With a similar approach two more circles can be added, as the second image from the left in Figure 2. The thick line shows the boundary of the motif that created the existing tiling. Another approach in constructing this motif, as is shown in the third image in Figure 2, is to select the 60° arc of *AB* (which is equal to 1/6 of the circle) and then apply a 180° rotation and two reflections to this arc. By the repetition of this motif, based on the lines of symmetry, which are shown in the right image in Figure 2, secondary frames are made in which the internal floral patterns are drawn.

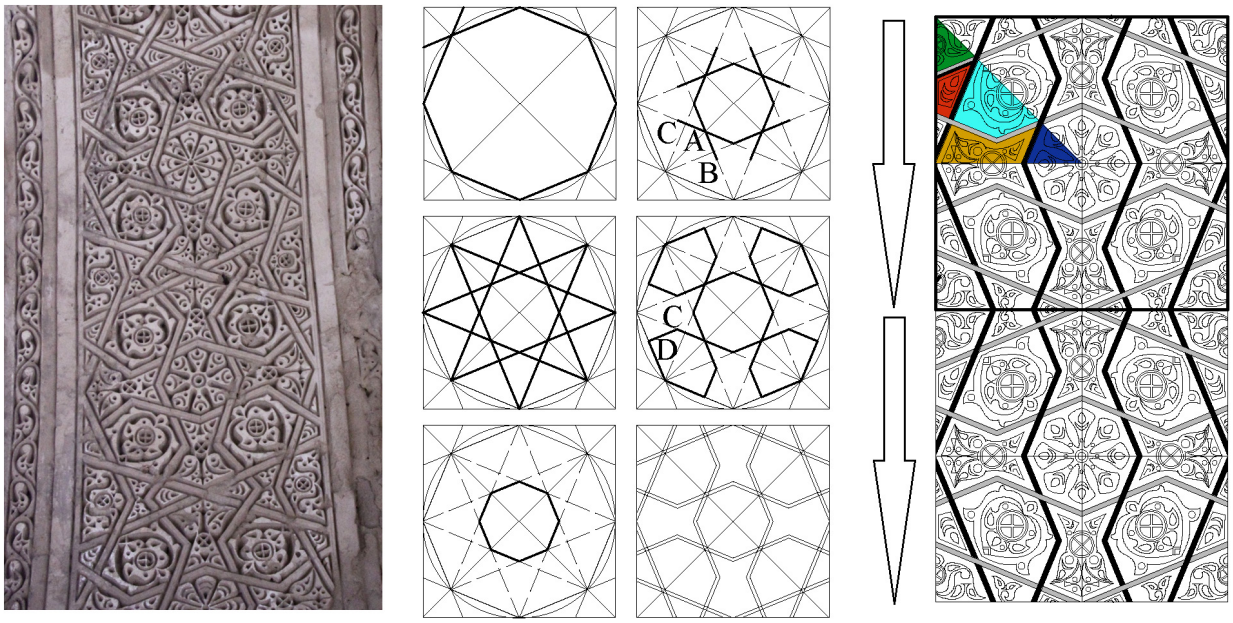


**Figure 3:** *Left, the stucco design in the eastern porch. Right, the analysis of its underlying design.*

Figure 3 illustrates a part of a wall, the design method and proliferation of motifs on this wall, and its underlying substructures. An octagon is inscribed inside a square and its sides extended. Then an (8, 3) star polygon, a figure created from connecting every third vertex in a set of eight equally spaced points on a circle in one direction using one stroke, is constructed inside. Now concentrate on the inner octagon, extend its sides as much as the length of each side (here we extend segment *AB* from its *A* side to find *CB*, where *A* becomes the midpoint of *CB*). Now from the free end points of these segments (here in our example *C*) make perpendicular segments to the sides of the big octagon (*CD*). Repeat this construction in other directions to complete the entire design. This stucco has rotational symmetry in one repetitive square unit. When we add interlocking strips to the pattern, we should consider the way that they should be interwoven. They have to go up and down from other strips based on a rotational symmetry about the center of each octagon. To complete the stucco pattern the generated motif unit should be transferred according to the vectors which are shown by arrows.

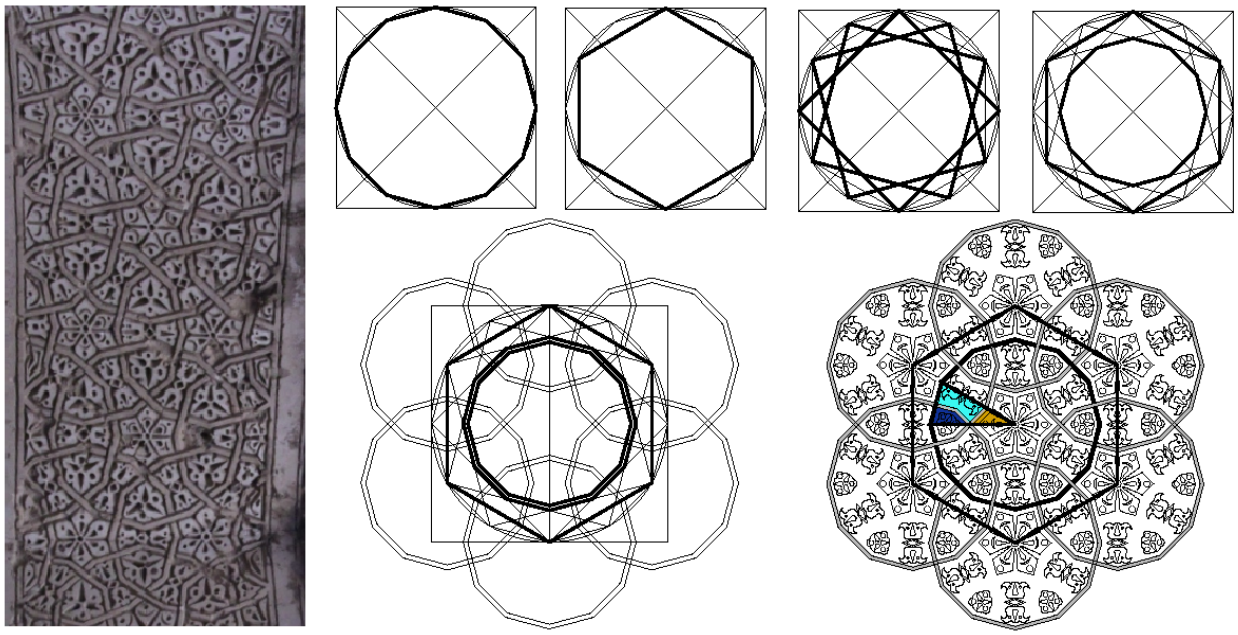

**Figure 4:** *Left: stucco in the entrance Iwan. Right: analysis of substructure.*

The stuccowork on a wall of the building in Figure 4 is created based on the repetition of a regular dodecagon, a 12-gon, about the vertices of a hexagon. As is illustrated at the top part of Figure 4, the underlying geometric construction can be made by drawing a square and inscribing a decagon inside of it. We then draw a hexagon inside of the dodecagon by connecting every other vertex. The next step is to connect every third vertex of the dodecagon by segments in one direction to create three inscribed squares. The intersections of these three square are the vertices of another dodecagon. This dodecagon is a repetitive unit for our construction and should be constructed about the vertices of the hexagon, as are shown in the middle image in Figure 4. Similar to the study of the previous tessellation in Figure 3 the interlocking interwoven strips should be added to the design. The details in this stuccowork exhibits the creative mind of its designer of many years ago who used simple tools such as straightedge and compass to demonstrate some proportional properties of regular hexagons and dodecagons and their relationships.

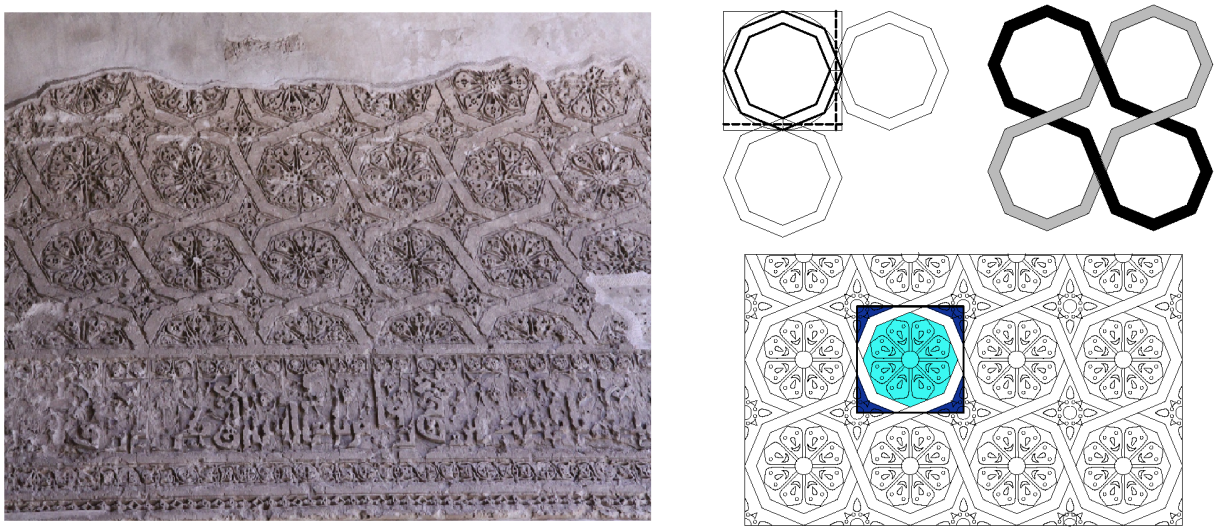

**Figure 5:** *Left, stucco in the southern iwan. Right, analysis of underlying design.*

For drawing the pattern appearing on a part of a wall in Forumad mosque, which is exhibited in the left image of Figure 5, one should inscribe an octagon inside of a square. The next step is to draw a smaller octagon inside of the original one to create necessary interlocking strips for this design. The entire pattern is created by reflecting the octagon based on vertical and horizontal lines of symmetry. As the rule, the strips should revolve up and down in all directions. The star forms that are created inside the octagons, as well as the art forms between them, complete the underlying design of the stuccowork.

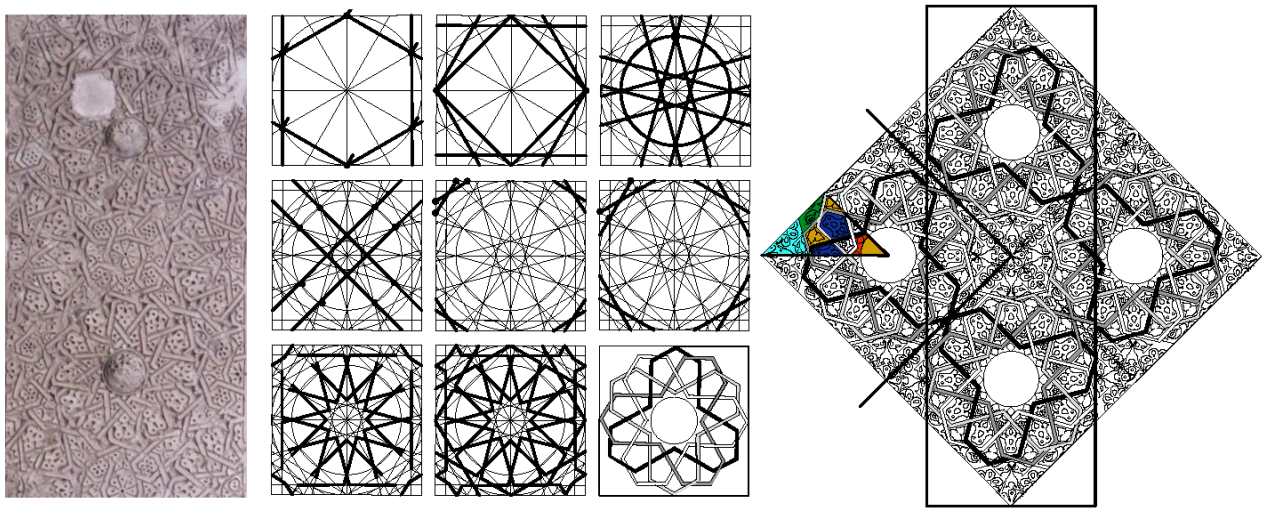

**Figure 6:** *Left: stucco under the northern Iwan. Right: analysis of substructure.*

The underlying design of the actual stuccowork in Figure 6 is performed in nine steps from top to bottom and from left to right, which is presented in the middle image of Figure 6. Similar to Figure 4 but perhaps more complex, we witness an extensive usage of compass and straightedge in the geometric construction of this design. The steps taken for construction of the motif are similar to the analysis that Broug presents for a Mamluk Quran illustration [2]. As is illustrated in the right image in Figure 6, the repetitive motif unit is a square that is reflected horizontally and vertically based on its sides. The main pattern has strips

that are revolved up and down. So, in order to have an accurate pattern the repetitive unit is transferred by the symmetry lines, which are shown. The final pattern, which includes interlocking strips, has rotational symmetry.

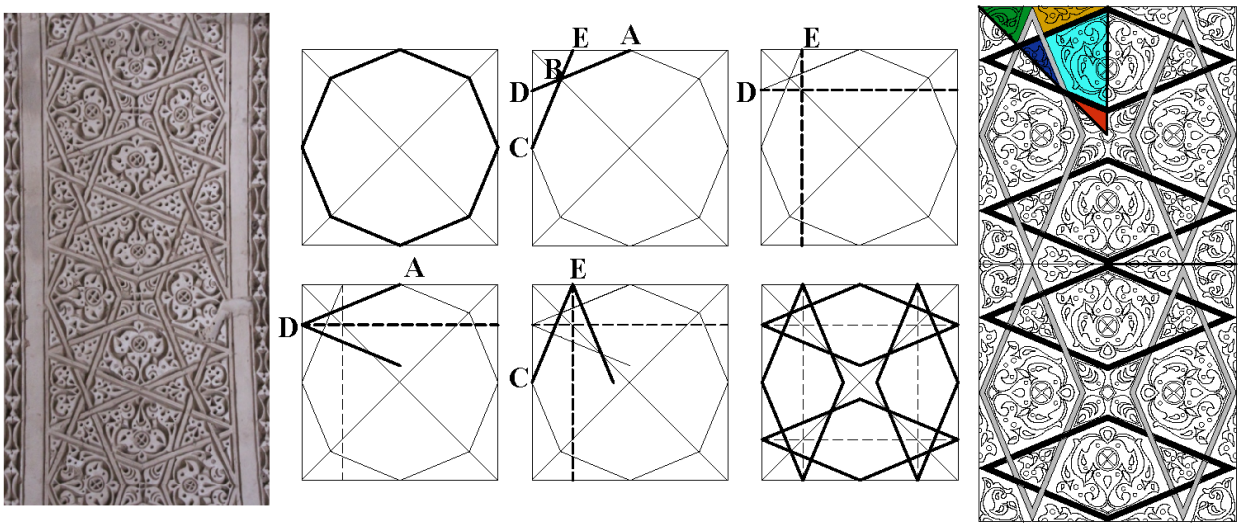


**Figure 7:** *Left, stucco and glazed brick, northern iwan. Right, Analysis of substructure.*

In Figure 7, the underlying design of the stuccowork of a wall on the northern iwan is studied. The existing pattern is shown in the left image. The six drawings in the middle show the way that we start with an octagon, which is inscribed in a square. The entire pattern is drawn based on the design of a one-forth of it and it is completed by its proliferation using vertical and horizontal symmetries. Extend the octagon's sides to meet the squares sides. As is demonstrated in the figure we extend segments $AB$ and $CB$, to find the points $D$ and $E$. Then we draw a horizontal line at $D$ to its opposite side of the square and transfer the segment $AD$ based on this line of symmetry. Again, we do this for the segment $CE$ as well. Finally, the underlying design is completed by proliferation of the one-forth part. In this pattern strips are also added. It is obvious that by transferring the repetitive unit based on the line of symmetry, which is shown, strips revolve up and down easily.

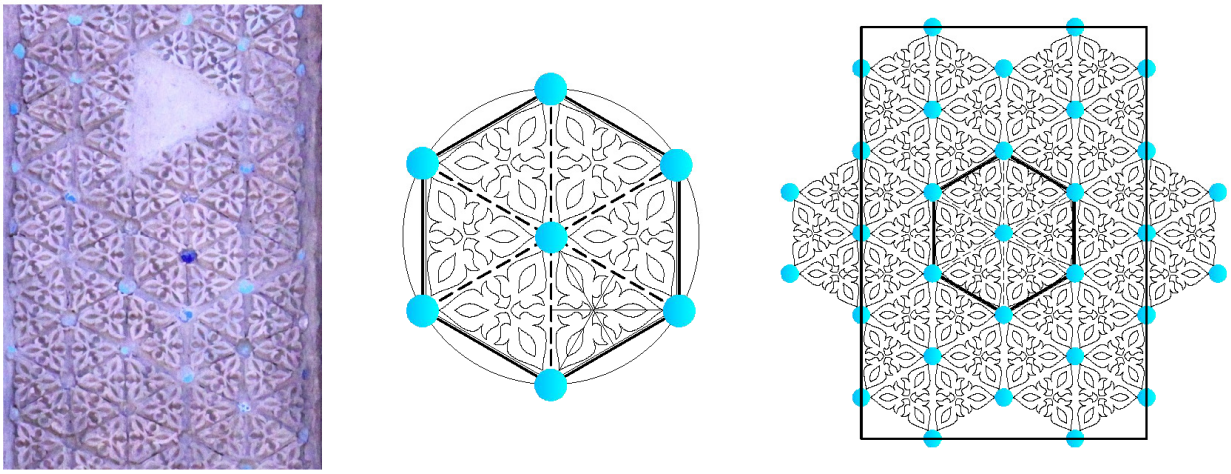

**Figure 8:** *Left: stucco and glazed brick, northern Iwan. Right: Analysis of sub structure.*

The underlying design on the existing stuccowork in the left image of Figure 8 is based on a hexagon. Draw a hexagon and connect its vertices properly to create six equilateral triangles. A motif is created in one of the triangles. By transferring this motif using rotational symmetry about the center of the hexagon, a hexagonal repetitive unit is created. As it is shown in the figure, the entire decorative panel is completed by the proliferation of the hexagonal repetitive unit using translations.

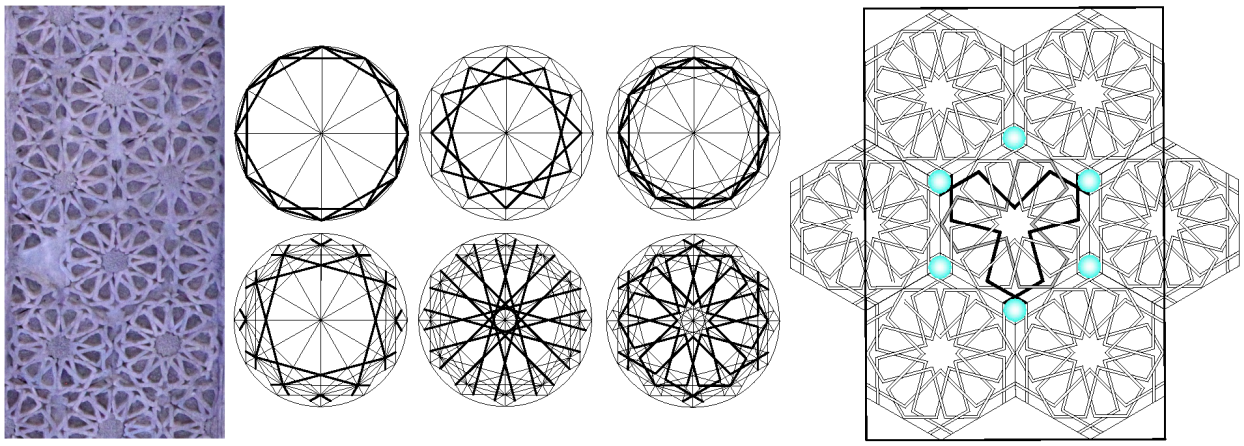

**Figure 9:** *Left, stucco and Glazed brick of the southern Iwan. Right, analysis of its underlying design.*

The underlying design of the stuccowork in Figure 9 is generated based on a hexagonal repetitive unit. The method for creating the motif for this design, which is borrowed from a book by El-Said [3], is as follows: Draw a circle and inscribe a dodecagon inside of it. Connect every second vertex of the dodecagon in one direction. This will generate two concentric congruent hexagons, as is illustrated in the first circle in the middle six images from top left to the bottom right. Consider the midpoints of the sides of the two inscribed hexagons. Connect every third vertex to make three concentric congruent squares. Use the segments created from the intersections of the hexagons and squares to make more segments and continue completing the pattern according to the illustrated images in the figure. The entire pattern, which is shown in the right image, is made from copies of the star presented in the last step of the middle construction. By extending the sides of the stars, the craftsmen made appropriate places for decorated glazed bricks. Unfortunately, most of these glazed bricks have been destroyed.

## 4 Conclusion

Forumad mosque is a prototype structure in applying widespread ornamental designs in Islamic architecture in Iran. Abstract floral forms used in the underlying designs that adorn the walls of this building have been constructed in accurate proportions. The ornaments exhibit extensive applications of geometric constructions. Early mosques were very simple with no decoration. Forumad is one of the earlier examples of Iranian mosques from the $12^{th}$ or early $13^{th}$ century, which shows the inception of applying ornaments to Islamic structures. In this article we tried to exhibit the designers' achievement in constructing the underlying geometric patterns. This certainly shows their knowledge of geometry and their fascination in designing architectural ornaments using straightedge and compass, the common instruments at that time.

Beside geometric patterns, there are some floral patterns in the ornaments of this mosque with their fluid lines being guided by the designers based on geometric substructures: So that there is not even a bit of heterogeneity or clutter in the entire decorative panels.

**References**

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